$$
\begin{aligned}
\Sigma_{IS} = & \left(-\frac{2}{\mathcal{H}_s r_s}-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\left\{-v_{||s}^{(2)}-\frac{1}{2}\phi_s^{(2)}-\frac{1}{2}\int_{\eta_s}^{\eta_0}d\eta'\partial_{\eta'}\left[\phi^{(2)}\left(\eta'\right)+\psi^{(2)}\left(\eta'\right)\right]+\frac{1}{2}\left(v_{||s}\right)^2\right. \\
& +\frac{1}{2}\left(\psi_s^I\right)^2+\left(-v_{||s}-\psi_s^I\right)\left(-\psi_s^I-2\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)\right)+\frac{1}{2}v_{\perp s}^a v_{\perp a\, s} \\
& +2a\, v_{\perp s}^a\partial_a\int_{\eta_s}^{\eta_o}d\eta'\psi^I\left(\eta'\right)+4\int_{\eta_s}^{\eta_0}d\eta'\left[\psi^I\left(\eta'\right)\partial_{\eta'}\psi^I\left(\eta'\right)+\partial_{\eta'}\psi^I\left(\eta'\right)\int_{\eta'}^{\eta_o}d\eta''\partial_{\eta''}\psi^I\left(\eta''\right)\right. \\
& \left.+\psi^I\left(\eta'\right)\int_{\eta'}^{\eta_o}d\eta''\partial_{\eta''}^2\psi^I\left(\eta''\right)-\gamma_0^{ab}\partial_a\left(\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)\right)\partial_b\left(\int_{\eta'}^{\eta_o}d\eta''\partial_{\eta''}\psi^I\left(\eta''\right)\right)\right] \\
& +2\partial_a\left(v_{||s}+\psi_s^I\right)\int_{\eta_s}^{\eta_o}d\eta'\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right) \\
& +4\int_{\eta_s}^{\eta_o}d\eta'\partial_a\left(\partial_{\eta'}\psi^I\left(\eta'\right)\right)\int_{\eta'}^{\eta_o}d\eta''\gamma_0^{ab}\partial_b\int_{\eta''}^{\eta_o}d\eta'''\psi^I\left(\eta'''\right)\Bigg\} \\
& +\left[\frac{1}{2}\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}+\frac{3}{2}\left(\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)^2-\frac{1}{2}\frac{\mathcal{H}_s''}{\mathcal{H}_s^3}+\frac{1}{\mathcal{H}_s r_s}\left(1+3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}+\frac{1}{\mathcal{H}_s r_s}\right)\right]\left[\left(v_{||s}\right)^2+\left(\psi_s^I\right)^2+2\psi_s^I v_{||s}\right. \\
& \left.+4\left(v_{||s}+\psi_s^I\right)\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)+4\left(\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)\right)^2\right]-\psi_s^{(2)}+\frac{1}{2}\phi_s^{(2)}+\frac{1}{2\mathcal{H}_s}\partial_\eta\psi_s^{(2)}
\end{aligned}
$$